

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2023

SECOND YEAR (BATCH 2021-23)

Date : 12/05/2023

Time : 11.00 am – 1.00 pm

MATHEMATICS

Paper : CC16

Full Marks : 50

Group:- A

Answer **any two** questions:

[2×10]

1. a) Find the Fourier transform of the function $f(x) = e^{-x^2/2}$.
- b) If the Fourier transform of $f(x)$ is $F(s)$, then find the Fourier transform of $e^{ibx} f(ax)$.
- c) Establish Parseval's relation for Fourier transforms

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds, \text{ where } F(s) \text{ is the Fourier transform of } f(x). \quad [3+3+4]$$

2. Use Fourier transform to solve the Laplace equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, -\infty < x < \infty, y \geq 0$,
with the boundary condition $\phi(x, 0) = f(x), -\infty < x < \infty$, $f(x)$ being a prescribed function
whose Fourier transform exists and $\phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \rightarrow 0$ as $\sqrt{x^2 + y^2} \rightarrow \infty$. [10]

3. a) Solve : $(D-2)x - (D+1)y = 6e^{3t}$

$$(2D-3)x + (D-3)y = 6e^{3t}$$

where $x=3$ and $y=0$ when $t=0$. (Use Laplace transformation)

b) Evaluate $L\left[\frac{\sinh t}{t}\right]$.

- c) State convolution theorem for Laplace transformation.

[6+2.5+1.5]

Group:- B

[All the notations have their usual meaning.]

Answer **all** the questions, maximum one can score is 15.

4. Find the eigenvalues and the corresponding eigen-functions of the integral equation

$$y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t) dt. \quad [6]$$

5. Using iterative method, solve $y(x) = f(x) + \lambda \int_0^1 e^{x-t} y(t) dt$. [6]

6. Solve the equation $y(t) = e^{-t} - 2 \int_0^t \cos(t-x) y(x) dx$ by using Laplace transform. [6]

Group:- C

[All the notations have their usual meaning.]

Answer **all** the questions, maximum one can score is 15.

7. Find the extremal of $\int_0^2 \frac{(y')^2}{x} dx$ with $y(0)=0, y(2)=1$. [5]
8. Find the extremal of the functional $\int_0^\pi ((y')^2 - y^2) dx$ under the conditions $y(0)=0, y(\pi)=1$ and subject to the condition $\int_0^\pi y dx = 1$. [6]
9. Find the shortest path from the point $P(-2,3)$ to the point $Q(2,3)$ located in the region $y \leq x^2$. [7]

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